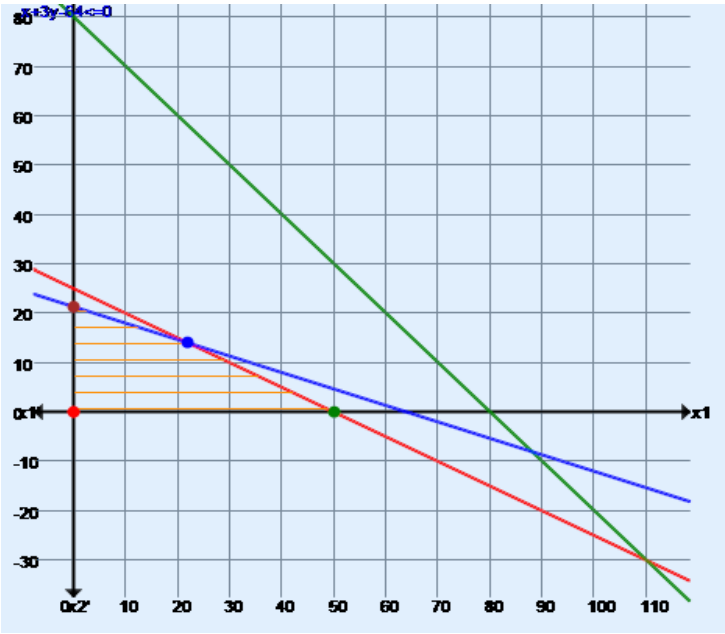
เฉลยบทที่ 1

1

1.1

```
O.F. Max 7x1+15x2

S.T.
     x1+2x2 <= 50
     x1+x2 <= 80
     x1+3x2 <= 64
     x1,x2 >= 0
```

1.2



1. To draw constraint $x_1 + 2x_2 \leq 50 \rightarrow (1)$

Treat it as $x_1 + 2x_2 = 50$

When $x_1 = 0$ then $x_2 = ?$

$\Rightarrow (0) + 2x_2 = 50$

$\Rightarrow 2x_2 = 50$

$\Rightarrow x_2 = \frac{50}{2} = 25$

When $x_2 = 0$ then $x_1 = ?$

$\Rightarrow x_1 + 2(0) = 50$

$\Rightarrow x_1 = 50$

x_1	0	50
x_2	25	0

2. To draw constraint $x_1 + x_2 \leq 80 \rightarrow (2)$

Treat it as $x_1 + x_2 = 80$

When $x_1 = 0$ then $x_2 = ?$

$\Rightarrow (0) + x_2 = 80$

$\Rightarrow x_2 = 80$

When $x_2 = 0$ then $x_1 = ?$

$\Rightarrow x_1 + (0) = 80$

$\Rightarrow x_1 = 80$

x_1	0	80
x_2	80	0

3. To draw constraint $x_1 + 3x_2 \leq 64 \rightarrow (3)$

Treat it as $x_1 + 3x_2 = 64$

When $x_1 = 0$ then $x_2 = ?$

$\Rightarrow (0) + 3x_2 = 64$

$\Rightarrow 3x_2 = 64$

$\Rightarrow x_2 = \frac{64}{3} = 21.33$

When $x_2 = 0$ then $x_1 = ?$

$\Rightarrow x_1 + 3(0) = 64$

$\Rightarrow x_1 = 64$

x_1	0	64
x_2	21.33	0

The value of the objective function at each of these extreme points is as follows:

Extreme Point Coordinates (x_1, x_2)	Lines through Extreme Point	Objective function value $z = 7x_1 + 15x_2$
$O(0, 0)$	$4 \rightarrow x_1 \geq 0$ $5 \rightarrow x_2 \geq 0$	$7(0) + 15(0) = 0$
$A(50, 0)$	$1 \rightarrow x_1 + 2x_2 \leq 50$ $5 \rightarrow x_2 \geq 0$	$7(50) + 15(0) = 350$
$B(22, 14)$	$1 \rightarrow x_1 + 2x_2 \leq 50$ $3 \rightarrow x_1 + 3x_2 \leq 64$	$7(22) + 15(14) = 364$
$C\left(0, \frac{64}{3}\right)$	$3 \rightarrow x_1 + 3x_2 \leq 64$ $4 \rightarrow x_1 \geq 0$	$7(0) + 15\left(\frac{64}{3}\right) = 320$

The maximum value of the objective function $z = 364$ occurs at the extreme point (22, 14).

Hence, the optimal solution to the given LP problem is : $x_1 = 22$, $x_2 = 14$ and max $z = 364$.

1.3

After introducing slack variables

$$\text{Max } Z = 7x_1 + 15x_2 + 0S_1 + 0S_2 + 0S_3$$

subject to

$$x_1 + 2x_2 + S_1 = 50$$
$$x_1 + x_2 + S_2 = 80$$
$$x_1 + 3x_2 + S_3 = 64$$

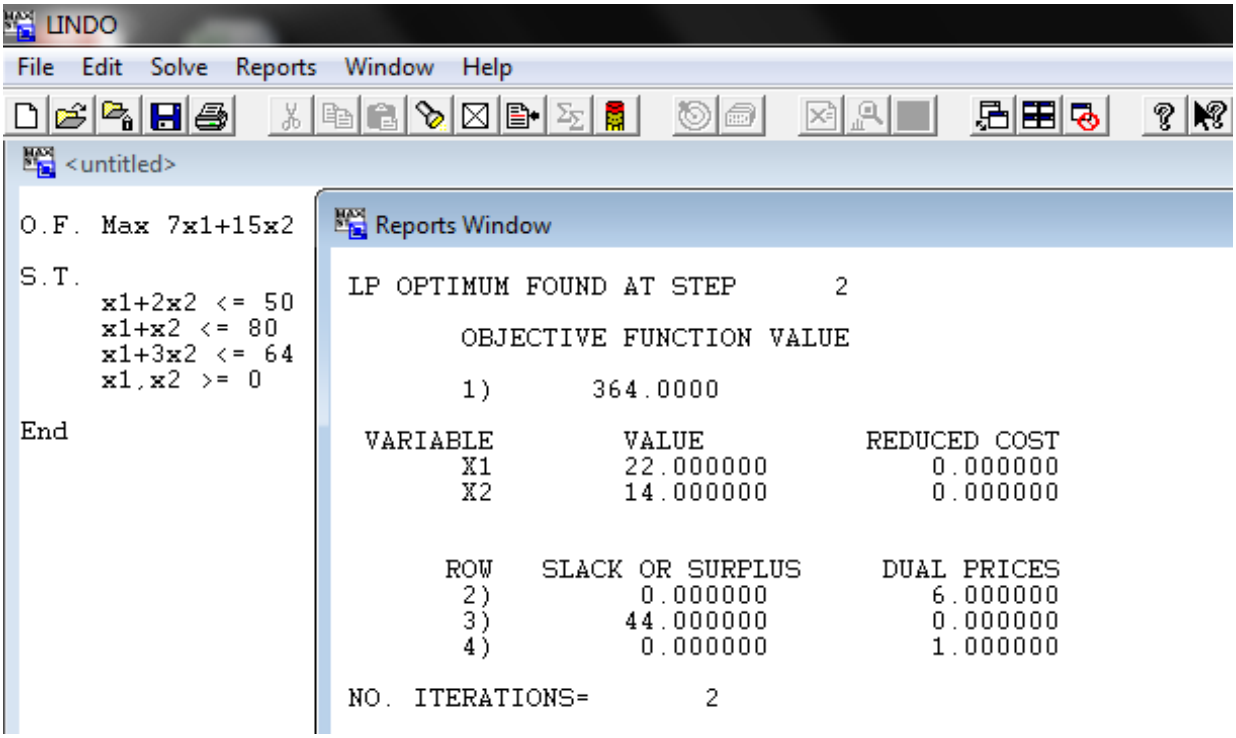
and $x_1, x_2, S_1, S_2, S_3 \geq 0$

Iteration-1		C_j	7	15	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{x_2}$
S_1	0	50	1	2	1	0	0	$\frac{50}{2} = 25$
S_2	0	80	1	1	0	1	0	$\frac{80}{1} = 80$
S_3	0	64	1	(3)	0	0	1	$\frac{64}{3} = \frac{64}{3} \rightarrow$
Z = 0		Z_j	**0**	**0**	**0**	**0**	**0**	
		$C_j - Z_j$	7	15 ↑	0	0	0	

Iteration-2		C_j	7	15	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio** $\frac{X_B}{x_1}$
S_1	0	$\frac{22}{3}$	$\left(\frac{1}{3}\right)$	0	1	0	$-\frac{2}{3}$	$\frac{\frac{22}{3}}{\frac{1}{3}} = 22 \rightarrow$
S_2	0	$\frac{176}{3}$	$\frac{2}{3}$	0	0	1	$-\frac{1}{3}$	$\frac{\frac{176}{3}}{\frac{2}{3}} = 88$
x_2	15	$\frac{64}{3}$	$\frac{1}{3}$	1	0	0	$\frac{1}{3}$	$\frac{\frac{64}{3}}{\frac{1}{3}} = 64$
Z = 320		Z_j	**5**	**15**	**0**	**0**	**5**	
		$C_j - Z_j$	2 ↑	0	0	0	-5	

Iteration-3		C_j	7	15	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	**MinRatio**
x_1	7	22	1	0	3	0	-2	
S_2	0	44	0	0	-2	1	1	
x_2	15	14	0	1	-1	0	1	
Z = 364		Z_j	**7**	**15**	**6**	**0**	**1**	
		$C_j - Z_j$	0	0	-6	0	-1	

1.4


LINDO
File Edit Solve Reports Window Help
<untitled>
O.F. Max 7x1+15x2
S.T.
x1+2x2 <= 50
x1+x2 <= 80
x1+3x2 <= 64
x1,x2 >= 0
End
Reports Window
LP OPTIMUM FOUND AT STEP 2
OBJECTIVE FUNCTION VALUE
1) 364.0000
VARIABLE VALUE REDUCED COST
X1 22.000000 0.000000
X2 14.000000 0.000000
ROW SLACK OR SURPLUS DUAL PRICES
2) 0.000000 6.000000
3) 44.000000 0.000000
4) 0.000000 1.000000
NO. ITERATIONS= 2

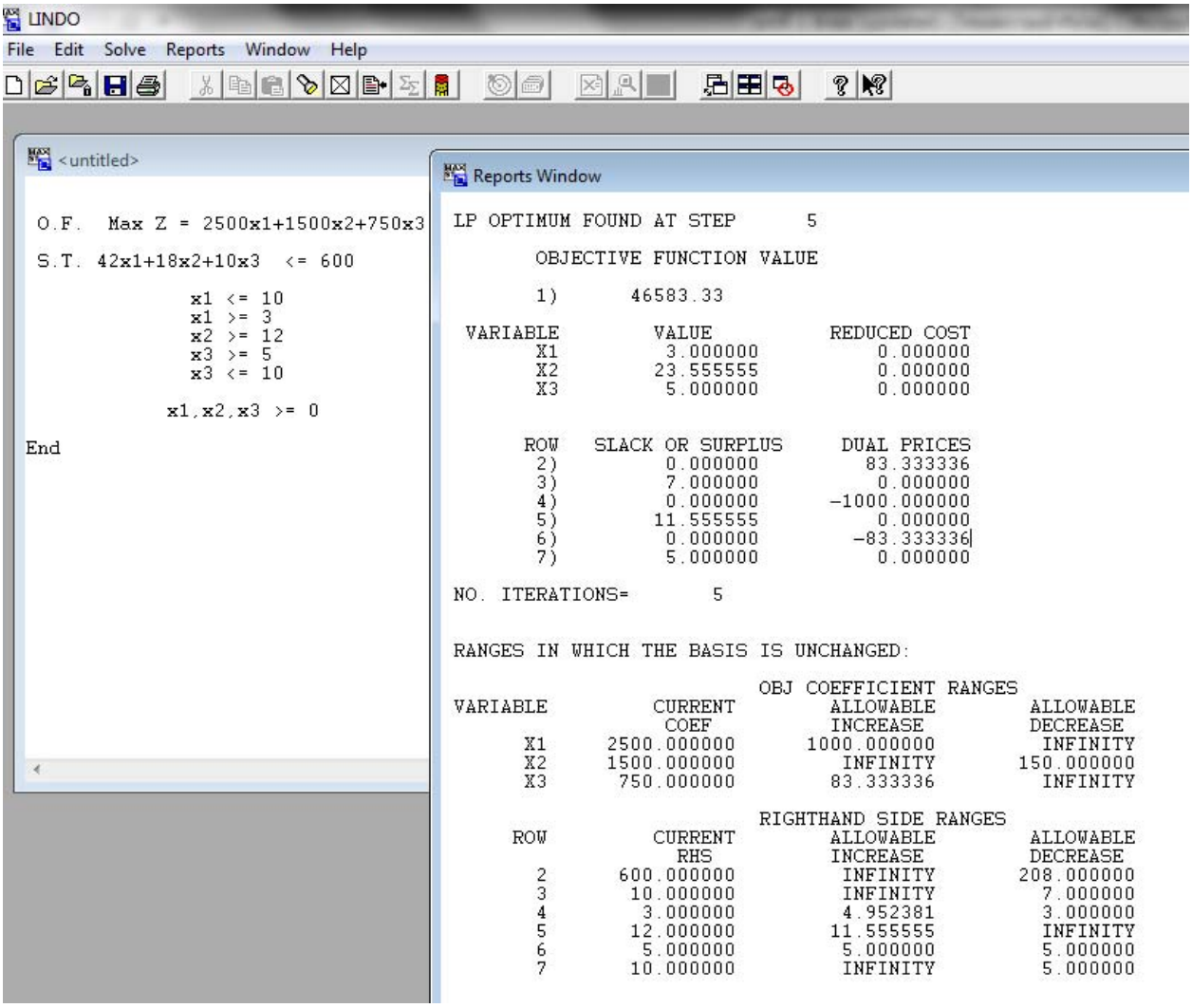
2

2.1

```
O.F.  Max Z = 2500x1+1500x2+750x3

S.T. 42x1+18x2+10x3  <= 600

            x1 <= 10
            x1 >= 3
            x2 >= 12
            x3 >= 5
            x3 <= 10

          x1,x2,x3 >= 0
```

2.2

2.3



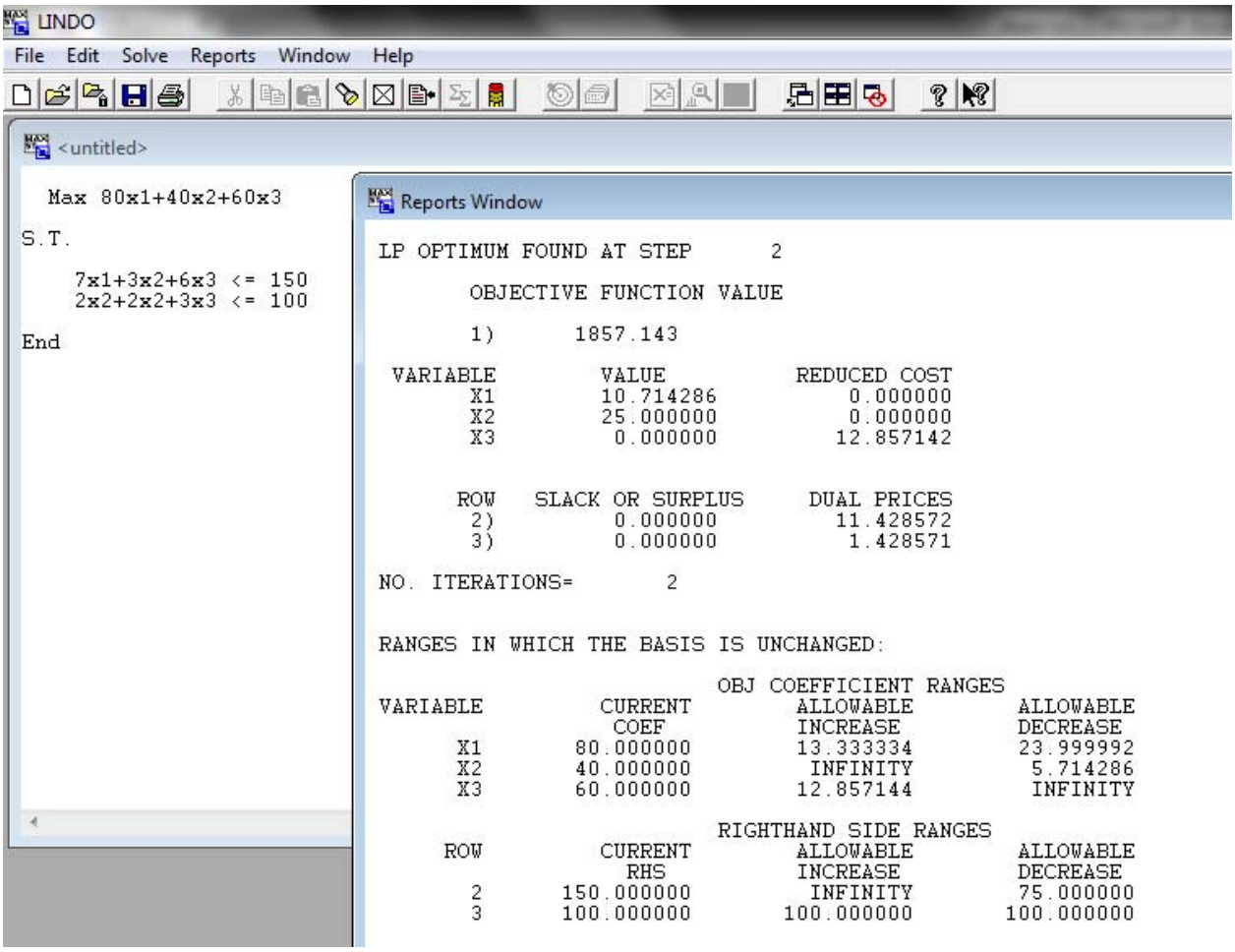
3

3.1

```
O.F. Max Z = 80x1+40x2+60x3

S.T.

    7x1+3x2+6x3 <= 150
    2x2+2x2+3x3 <= 100

    x1,x2,x3 >= 0
```

3.2

3.3

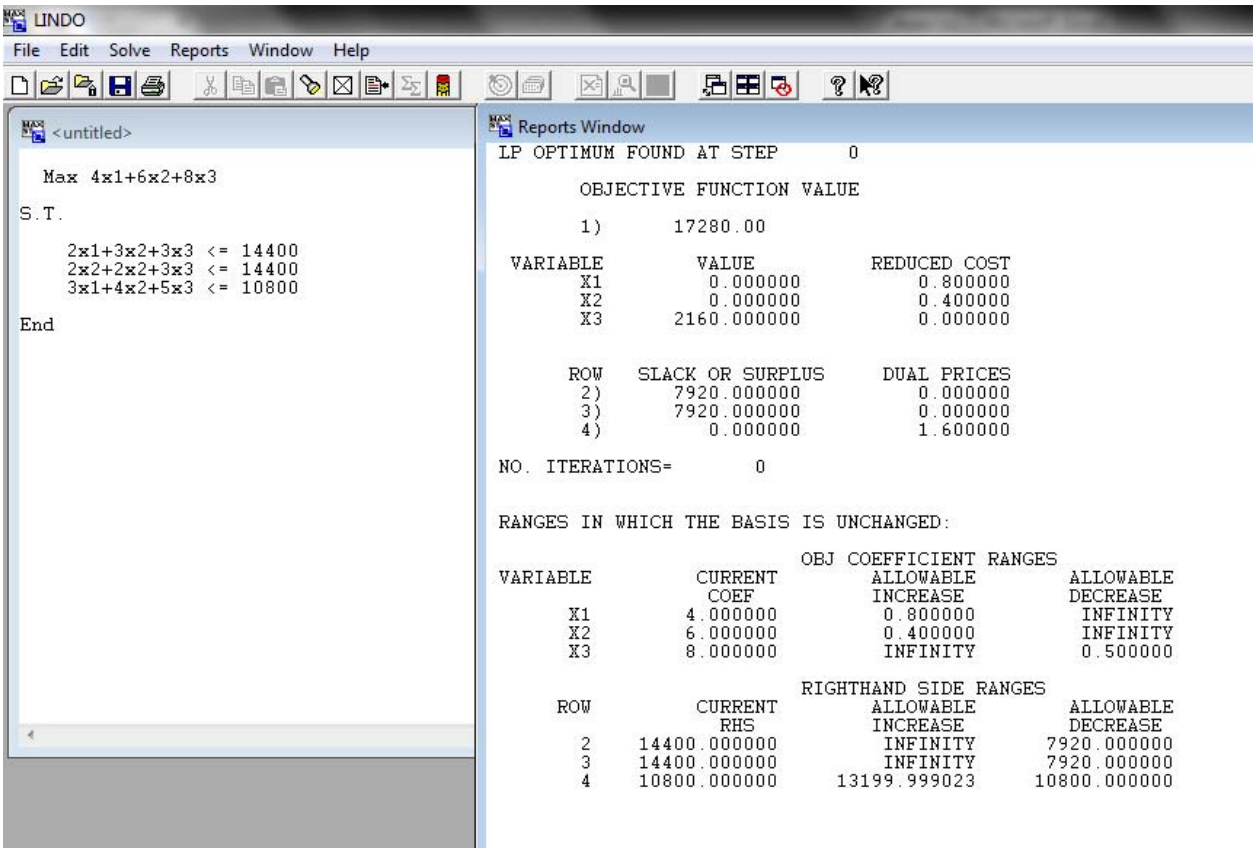


4

4.1

4.2

4.3


LINDO
File Edit Solve Reports Window Help
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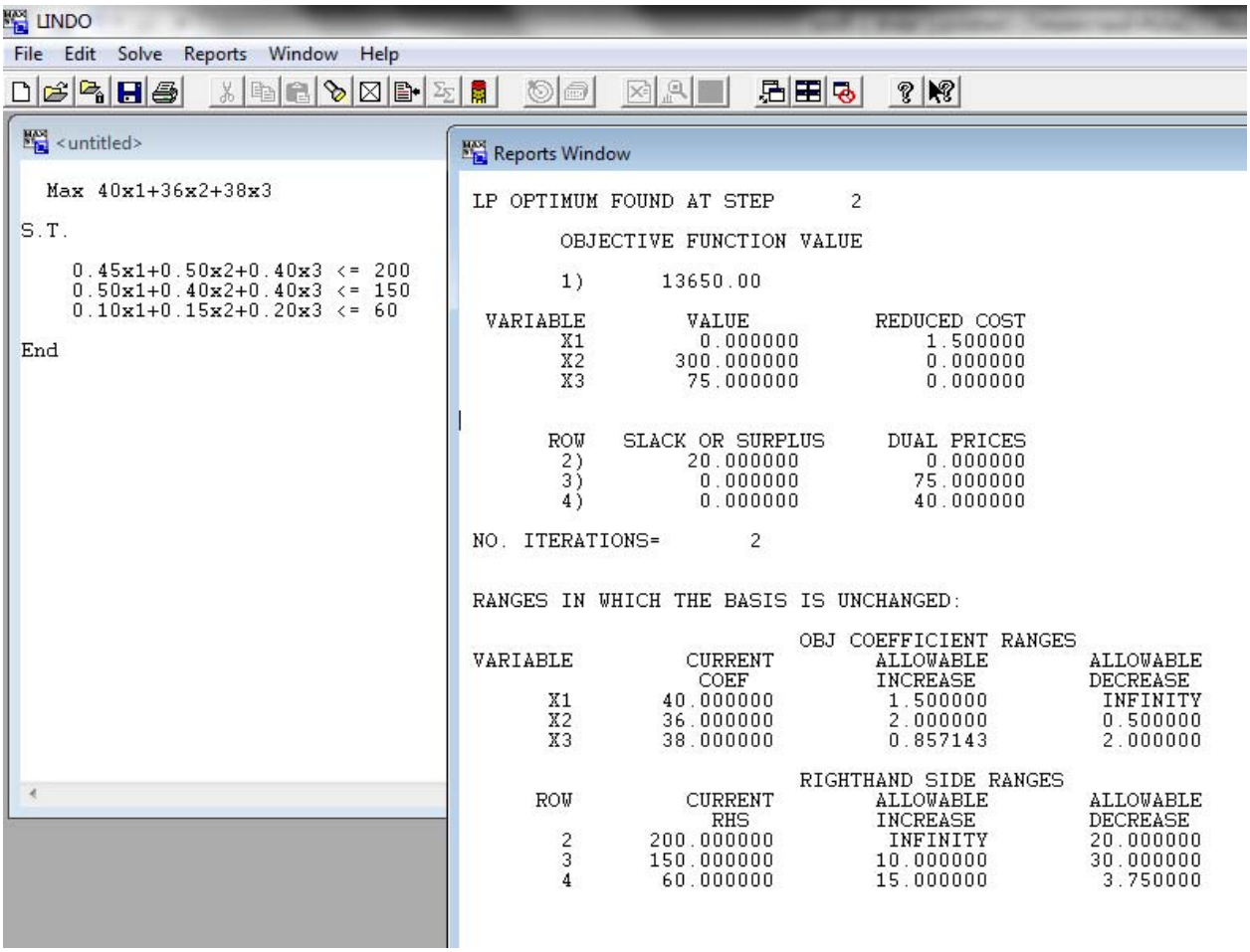
Max 4x1+6x2+8x3
S.T.
2x1+3x2+3x3 <= 14400
2x2+2x2+3x3 <= 14400
3x1+4x2+5x3 <= 10800
End
Reports Window
LP OPTIMUM FOUND AT STEP 0
OBJECTIVE FUNCTION VALUE
1) 17280.00
VARIABLE VALUE REDUCED COST
X1 0.000000 0.800000
X2 0.000000 0.400000
X3 2160.000000 0.000000
ROW SLACK OR SURPLUS DUAL PRICES
2) 7920.000000 0.000000
3) 7920.000000 0.000000
4) 0.000000 1.600000
NO. ITERATIONS= 0
RANGES IN WHICH THE BASIS IS UNCHANGED:
OBJ COEFFICIENT RANGES
VARIABLE CURRENT COEF ALLOWABLE INCREASE ALLOWABLE DECREASE
X1 4.000000 0.800000 INFINITY
X2 6.000000 0.400000 INFINITY
X3 8.000000 INFINITY 0.500000
RIGHTHAND SIDE RANGES
ROW CURRENT RHS ALLOWABLE INCREASE ALLOWABLE DECREASE
2 14400.000000 INFINITY 7920.000000
3 14400.000000 INFINITY 7920.000000
4 10800.000000 13199.999023 10800.000000

5

5.1

5.2

5.3

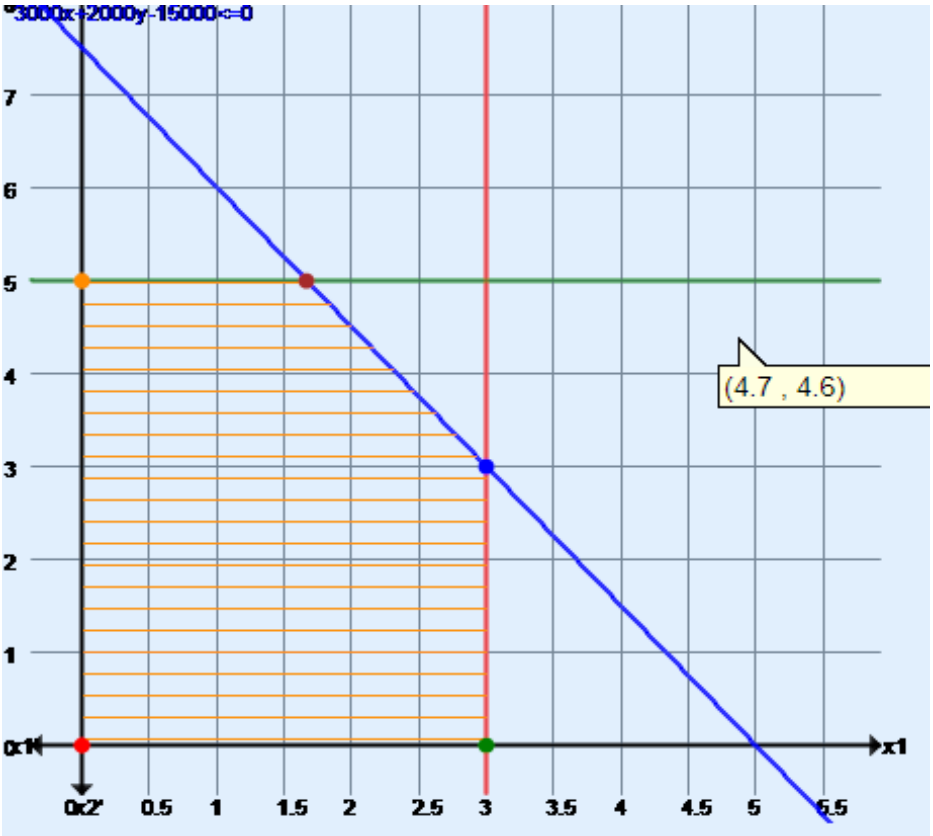

LINDO
File Edit Solve Reports Window Help
<untitled>
Max 40x1+36x2+38x3
S.T.
0.45x1+0.50x2+0.40x3 <= 200
0.50x1+0.40x2+0.40x3 <= 150
0.10x1+0.15x2+0.20x3 <= 60
End
Reports Window
LP OPTIMUM FOUND AT STEP 2
OBJECTIVE FUNCTION VALUE
1) 13650.00
VARIABLE VALUE REDUCED COST
X1 0.000000 1.500000
X2 300.000000 0.000000
X3 75.000000 0.000000
ROW SLACK OR SURPLUS DUAL PRICES
2) 20.000000 0.000000
3) 0.000000 75.000000
4) 0.000000 40.000000
NO. ITERATIONS= 2
RANGES IN WHICH THE BASIS IS UNCHANGED:
OBJ COEFFICIENT RANGES
VARIABLE CURRENT COEF ALLOWABLE INCREASE ALLOWABLE DECREASE
X1 40.000000 1.500000 INFINITY
X2 36.000000 2.000000 0.500000
X3 38.000000 0.857143 2.000000
RIGHTHAND SIDE RANGES
ROW CURRENT RHS ALLOWABLE INCREASE ALLOWABLE DECREASE
2 200.000000 INFINITY 20.000000
3 150.000000 10.000000 30.000000
4 60.000000 15.000000 3.750000

6

```
MAX z = 700x1 + 400x2
subject to
x1 <= 3
x2 <= 5
3000x1 + 2000x2 <= 15000
and x1,x2 >= 0
```

6.1

6.2



1. To draw constraint $x_1 \le 3 \rightarrow (1)$

Treat it as $x_1 = 3$

Here line is parallel to Y-axis

x_1	3	3
x_2	0	1

2. To draw constraint $x_2 \le 5 \rightarrow (2)$

Treat it as $x_2 = 5$

Here line is parallel to X-axis

x_1	0	1
x_2	5	5

3. To draw constraint $3000x_1 + 2000x_2 \le 15000 \rightarrow (3)$

Treat it as $3000x_1 + 2000x_2 = 15000$

When $x_1 = 0$ then $x_2 = ?$

$\Rightarrow 3000(0) + 2000x_2 = 15000$

$\Rightarrow 2000x_2 = 15000$

$\Rightarrow x_2 = \dfrac{15000}{2000} = 7.5$

When $x_2 = 0$ then $x_1 = ?$

$\Rightarrow 3000x_1 + 2000(0) = 15000$

$\Rightarrow 3000x_1 = 15000$

$\Rightarrow x_1 = \dfrac{15000}{3000} = 5$

x_1	0	5
x_2	7.5	0

The value of the objective function at each of these extreme points is as follows:

Extreme Point Coordinates (x_1, x_2)	Lines through Extreme Point	Objective function value $z = 700x_1 + 400x_2$
$O(0, 0)$	$4 \rightarrow x_1 \ge 0$ $5 \rightarrow x_2 \ge 0$	$700(0) + 400(0) = 0$
$A(3, 0)$	$1 \rightarrow x_1 \le 3$ $5 \rightarrow x_2 \ge 0$	$700(3) + 400(0) = 2100$
$B(3, 3)$	$1 \rightarrow x_1 \le 3$ $3 \rightarrow 3000x_1 + 2000x_2 \le 15000$	$700(3) + 400(3) = 3300$
$C\left(\frac{5}{3}, 5\right)$	$2 \rightarrow x_2 \le 5$ $3 \rightarrow 3000x_1 + 2000x_2 \le 15000$	$700\left(\frac{5}{3}\right) + 400(5) = \frac{9500}{3}$
$D(0, 5)$	$2 \rightarrow x_2 \le 5$ $4 \rightarrow x_1 \ge 0$	$700(0) + 400(5) = 2000$

The maximum value of the objective function $z = 3300$ occurs at the extreme point (3, 3).

Hence, the optimal solution to the given LP problem is : $x_1 = 3$, $x_2 = 3$ and max $z = 3300$.

7

Hence, optimal solution is arrived with value of variables as :

$x_1 = 0, x_2 = \dfrac{4}{3}, x_3 = \dfrac{9}{2}$

Min $Z = -\dfrac{25}{6}$

8

Hence, optimal solution is arrived with value of variables as :
$x_1 = 5, x_2 = 0, x_3 = 2, x_4 = 1$

Max $Z = 8$

9

Hence, optimal solution is arrived with value of variables as :
$x_1 = 200, x_2 = 0, x_3 = 100$

Max $Z = 700$

10

Hence, optimal solution is arrived with value of variables as :
$x_1 = 10, x_2 = 5$

Min $Z = 20$

11

Hence, optimal solution is arrived with value of variables as :
$x_1 = \frac{68}{9}, x_2 = \frac{8}{9}, x_3 = \frac{4}{3}$

Min $Z = \frac{52}{9}$

12

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 0, x_3 = 40$

Min $Z = 1200$